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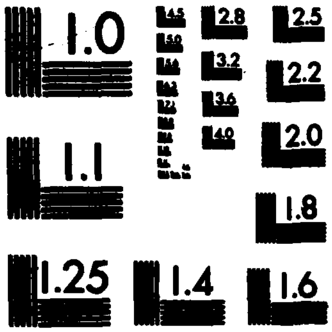

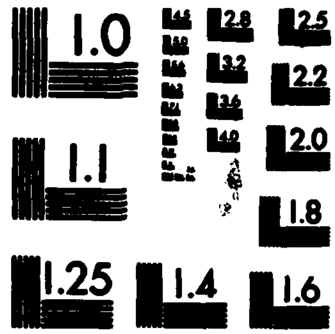

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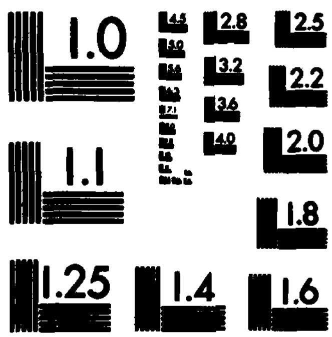

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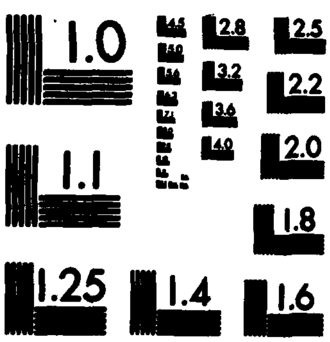

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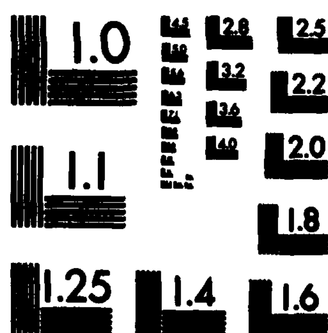

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19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Linear complementarity problem; parametric linear program; algorithms; worst case computational complexity; slope changes; NP-Complete class; NP-hard class; nearest point problem; edge covering problem; 1-matching/covering problem; adjacency checking; ellipsoid method; convex quadratic program.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This is a summary of research carried out under this grant. Some of the results are: for linear complementarity problems, existing pivot algorithms like Lemke's complementary pivot method, Murty's principal pivoting method, Cottle-Dantzig principal pivoting method, Vander Heyden's variable dimension algorithm, are all exponential growth algorithms in the worst case even on very nice classes of problems. Polynomially bounded ellipsoid methods have been developed for solving convex quadratic programs or linear complementarity problems associated with positive semidefinite matrices. A practically efficient critical index (CONT.)

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~~ITEM #20, CONTINUED:~~ algorithm based on orthogonal projections has been developed for linear complementarity problems associated with positive definite symmetric matrices. Efficient blossom algorithms have been developed for edge covering problems and 1-matching/covering problems in undirected networks, as well as efficient subroutines for performing various types of sensitivity analysis. A computer package for 1-matching/covering algorithms suitable for large scale applications has been developed. A bounding strategy for the set covering problem based on the 1-matching/covering problem and Lagrangean relaxation has been developed. Efficient linear time algorithms have been developed for checking the adjacency of two given 1-matching/covering incidence vectors on the 1-matching/covering polytope. The problem of checking the adjacency of two given 0-1 feasible solutions on the 0-1 polytope associated with many NP-Hard 0-1 Combinatorial optimization problems has been shown to be NP-Complete. The general inear complementarity problem has been shown to be strongly NP-Complete. Efficient methods have been developed for checking the convexity of a piecewise linear function defined on a convex polyhedron. In many parametric cost 0-1 integer and network flow problems, the number of slope changes in the optimum objective value function has been shown to grow exponentially with the size of the problem. Perturbation results were obtained for linear complementarity problems associated with positive semidefinite matrices.

FINAL SCIENTIFIC REPORT - SEPTEMBER, 1978 TO AUGUST, 1982

OF GRANT NO. AFOSR 78-3646

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RESEARCH OBJECTIVES.

The emphasis of the research is on the development of efficient algorithms for solving linear programming, linear complementarity and combinatorial optimization problems; on the study of the computational complexity of these algorithms and problems; and on the study of the geometric structure of these problems.

LIST OF PROFESSIONAL PERSONNEL WHO WORKED ON THIS PROJECT.

1. Professor Katta G. Murty, Principal Investigator.
2. Dr. Yahya Fathi, Research Assistant.
3. Dr. S. J. Chung, Research Assistant.
4. Dr. Clovis Perin, Research Assistant.
5. Ms. P. Carstensen, Research Assistant.
6. Mr. A. Gana, Research Assistant.
7. Mr. M. Partovi, Research Assistant.

SUMMARY OF RESEARCH ACCOMPLISHMENTS.

1. COMPUTATIONAL COMPLEXITY OF ALGORITHMS FOR LINEAR COMPLEMENTARITY PROBLEM (LCP).

An LCP of order n is a problem of the form: find $w \in R^n$, $z \in R^n$ satisfying

$$\begin{aligned} w - Mz &= q \\ w \geqq 0,\ z &\geqq 0 \qquad (1) \\ w^T z &= 0 \end{aligned}$$

where q, M are given matrices of orders n x 1 and n x n respectively. The first work in this area is a paper by K. G. Murty in which he constructed a class of LCPs of orders $n \geqq 2$, associated with lower triangular P-matrices (which are positive semidefinite, but not positive definite), and established that either Lemke's complementary pivot method or Murty's least index principal pivoting method require 2^n pivot steps to solve the nth problem in the class, before termination.

In [2] Y. Fathi established that these two algorithms exhibit the same exponential growth computational requirements in the worst case even on LCPs associated with positive definite symmetric matrices M.

In [11] J. Birge and A. Gana established that the Cottle-Dantzig principal pivoting algorithm, and the Van der Heyden variable dimension algorithm both exhibit exponential growth computational requirments in the worst case on LCPs associated with P-matrices M.

2. RESULTS ON LCPs ASSOCIATED WITH P-MATRICES

Let M be a P-matrix of order n and $J \subset \{1,\ldots n\}$. Consider the generalized LCP denoted by (q, M, J): find $w \in R^n$, $z \in R^n$ satisfying

$$
\begin{aligned}
& w - Mz = q \\
& w^T z = 0 \\
& w_j,\ z_j \geqq 0 \quad \text{for all } j \notin J \qquad (2) \\
& \qquad\ \leqq 0 \quad \text{for all } j \in J
\end{aligned}
$$

In [1] K. G. Murty has proved that (q, M, J) has a unique solution for each $q \in R^n$, iff M is a P-matrix.

Assume that q is nondegenerate in (1) and that M is a P-matrix. In [1] K. G. Murty has proved that given any +, - sign vector in R^n, there exists a unique complementary basis for (1) such that the components in the updated q with respect to it correspond to the specified sign vector. This establishes a 1-1 correspondence between complementary bases for (1) and sign vectors in R^n.

3. COMPLEXITY OF THE LCP.

In [5] S. J. Chung has proved that the general LCP is strongly NP-complete.

4. THE NEAREST POINT ALGORITHM.

Let A be a nonsingular square matrix of order n and b a given point in R^n. For finding the nearest point (in terms of the usual Euclidean distance) in the simplicial cone Pos(A) to b, K. G. Murty and Y. Fathi [4] have developed an efficient algorithm. This algorithm can also be used to solve LCPs associated with positive definite symmetric matrices.

5. COMPUTATIONAL COMPLEXITY OF PARAMETRIC LINEAR PROGRAMMING.

In [3], K. G. Murty has shown that the total number of slope changes in the optimum objective value function of a parametric linear program grows exponentially with the problem size in the worst case.

6. EFFICIENT BLOSSOM ALGORITHM FOR EDGE COVERING PROBLEMS.

Let $G = (N, A)$ be a connected undirected network with $c = (c_{ij})$ as the vector of edge costs. A subset of edges $E \subset A$ is an *edge cover* in G if for each $i \in N$, there exists at least one edge in E incident at node i.

In [6], K. G. Murty and C. Perin developed an efficient blossom algorithm for the problem of finding a minimum cost edge cover in G. Its worst case computational complexity is $O(n^3)$, where n is the number of nodes in G.

In [8], K. G. Murty and C. Perin devleoped an efficient lower bounding strategy for the minimum objective value in set covering problems, using the edge covering algorithm. This strategy can be used to develop efficient branch and bound algorithms for the set covering problem.

7. THE 1-MATCHING/COVERING PROBLEM.

Let $G = (N, A)$ be an undirected network with n nodes and m edges, with $c = (c_{ij})$ as the edge cost vector. Let $N^{\leq}, N^{=}, N^{\geq}, N^{0}$ be a given partition of the set of nodes N. A 1-matching/covering with respect to this partition is a subset of edges $F \subset A$ satisfying the property that for each $i \in N$; there is at most one edge in F incident at i if $i \in N^{\leq}$, exactly one edge if $i \in N^{=}$, and at least one edge if $i \in N^{\geq}$. The 1-matching/covering problem is to find a minimum cost 1 matching/covering. In this Ph.D. dissertation [23] and in [16],

C. Perin extended the blossom algorithm of [6] into an efficient algorithm for solving the 1-matching/covering problem, whose worst case computational complexity is $O(n^3)$. He has also developed another blossom algorithm of the out-of-kilter type in [23] for solving the 1-matching/covering problem, which also has the worst case computational complexity of $O(n^3)$. Combining these two algorithms, he has developed very effficient routines for sensitivity analysis in the 1-matching/covering problem, when changes occur in the edge cost vector, or the node partition, or when new edges or nodes are introduced, or when edges or nodes are eliminated.

C. Perin has prepared a software package in Fortran for these algorithms for the 1-matching/covering problem which has been tested extensively, and found to perform very well on even large networks. [17] is a user's guide for using this package, which contains a complete listing in an appendix.

In [15], C. Perin describes a simple proof for the blossom constraints characterizing the convex hull of the 1-matching/covering incidence vectors, which does not depend on the algorithm.

8. SPECIFIED CARDINALITY 1-MATCHING/COVERING PROBLEM.

This is the 1-matching/covering problem with an additional constraint on its cardinality r. It is required to solve this problem treating r as an integer valued parameter. In [18], K. G. Murty and C. Perin describes a blossom algorithm for this problem, whose worst case computational complexity is $O(n\ m^2)$.

9. POLYNOMIALLY BOUNDED ELLIPSOID ALGORITHMS FOR CONVEX QUADRATIC PROGRAMMING.

In [7], S. J. Chung and K. G. Murty have developed polynomially bounded ellipsoid algorithms for convex quadratic programs, or equivalently LCPs associated with positive semidefinite matrices, in which all the data is integer.

Y. Fathi [13] did a computational study of several algorithms for solving LCPs associated with positive definite symmetric matrices. In these computational tests he found out that the critical index algorithm discussed in [4] is the most efficient for solving this class of problems. The ellipsoid method of [7] performed very poorly in comparison with the others. These results confirm the observatin that, while the ellipsoid method is a tremendous theoretical breakthrough, it is not very practical for solving linear or convex quadratic programming problems.

10. COMPLEXITY OF ADJACENCY CHECKING.

In [19, 24], S. J. Chung has developed efficient linear time algorithms for checking whether two given 1-matching/covering incidence vectors are adjacent on the convex hull of all 1-matching/covering incidence vectors.

In [20, 24], S. J. Chung has studied the problem of checking the adjacency of two given 0-1 feasible solutions on the convex hull of all 0-1 feasible solutions of the following problems (i) the 0-1 equality constrained Knapsack problem, (ii) the 0-1 inequality constrained Knapsack problem, (iii) the general 0-1 integer programming problem, (iv) the general set covering problem. In each case, he has developed necessary and sufficient conditions for two given 0-1 feasible solutions to be adjacent on the convex hull of all 0-1 feasible solutions of that problem.

But unfortunately, it turns out that checking whether these conditions hold, is an NP-complete problem in each case. This conclusively establishes that the problem of adjacency checking of two given extreme points on the integer polytope associated with each these problems in NP-complete.

In [20, 24], S. J. Chung has also established that the problem of checking whether two given extreme points of an unbounded convex polyhedron are adjacent on the convex hull of all extreme points of this polyhedron, is itself an NP-complete problem. A special case of this, to check whether two given simple chain incidence vectors are adjacent on the convex hull of all simple chain incidence vectors, is also an NP-complete problem.

11. PERTURBATION RESULTS FOR LCPs ASSOCIATED WITH POSITIVE SEMIDEFINITE MATRICES.

Consider an LCP (q, M) in which M is positive semidefinite. Consider the perturbed LCP $(q, M + \varepsilon I)$ where $\varepsilon > 0$. In [21], A. Gana has derived the limiting behavior of the solution of the perturbed problem as ε converges to zero.

12. COMPLEXITY OF PARAMETRIC INTEGER AND NETWORK FLOW PROBLEM.

In [12], P. Carstensen has studied the behavior of the number of slope changes in the optimum objective function of parametric cost single commodity network flow problem and parametric cost 0-1 integer programming problems. There, she has constructed a parametric cost minimum cost flow problem with $2n + 2$ nodes, $O(n^2)$ arcs, with $O(2^n)$ slope changes in the optimum objective value function, for each positive integer n. Similarly she has constructed parametric cost 0-1 integer programming problems (even Knapsack problems) in which the number of slope changes for the optimum objective function value grows as $2^{\sqrt{n}}$ where n is the number of 0-1 variables in the problem. This clearly establishes that in both these classes of parametric cost problems, the number of slope changes in the optimum objective value curve grows exponentially with the size of the problem.

13. SOME NP-COMPLETE PROBLEMS IN LINEAR PROGRAMMING.

In [10], R. Chandrasekharan, S. N. Kabadi and K. G. Murty have established that many of the unresolved problems in linear and quadratic programming (for example, the problem of checking whether an LP with integer data is degenerate, whether there exists a basic feasible solution for a given LP with a specified objective value, whether a given square matrix has a singular principal submatrix, the bilinear programming problem, etc.) are either NP-complete or NP-hard.

14. CONVEXITY OF PIECEWISE LINEAR FUNCTIONS.

In [14], K. G. Murty has established the necessary and sufficient conditions for a given piecewise linear function on a convex polyhedron, to be convex. He has developed an algorithm for checking whether the given piecewise linear function is convex, which requires the solution of a small number of linear programs.

PUBLISHED PAPERS WRITTEN UNDER THE PROJECT.

1. K. G. Murty, "On the linear complementarity problem", pp. 425-439, *Methods of Operations Research*, Band 31, Athenaum/Hain/Scriptor/Hanstein, 1978.

2. Y. Fathi, "Computational complexity of LCPs associated with positive definite symmetric matrices", *Mathematical Programming*, 17, pp. 335-344, 1979.

3. K. G. Murty, "Computational complexity of parametric linear programming", *Mathematical Programming*, 19, pp. 213-219, 1980.

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5. S. J. Chung, "A note on the complexity of LCP: The LCP is NP-complete", to appear in *Mathematical Programming*,

6. K. G. Murty and C. Perin, "A 1-matching blossom type algorithm for edge covering problems". To appear in *Networks*.

7. S. J. Chung and K. G. Murty, "Polynomially bounded ellipsoid algorithms for convex quadratic programming", pp. 439-485, in *Nonlinear Programming 4*, Academic Press, 1981.

8. K. G. Murty and C. Perin, "Edge covering algorithms and their applications", pp. 379-383, *Methods of Operations Research*, 40, Oelgeschlager, Gunn and Hain Inc., Cambridge, Massachusetts, 1981.

9. K. G. Murty, "Nonlinear optimization", pp. 14.3.1 to 14.3.19, in *Handbook of Industrial Engineering*, G. Salvendy (editor), Wiley-Interscience, 1982.

10. R. Chandrasekharan, S. N. Kabadi and K. G. Murty, "Some NP-complete problems in linear programming", *Operations Research Letters*, *1*, *3*, pp. 101-104, July 1982.

11. J. R. Birge and A. Gana, "Computational complexity of Van der Heyden's variable dimension algorithm and Dantzig-Cottle's principal pivoting method for solving LCP's ". To appear in *Mathematical Programming*.

12. P. J. Carstensen, "Complexity of some parametric integer and network programming problems". To appear in *Mathematical Programming*.

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13. Y. Fathi, "A comparative study of the ellipsoid algorithm and other algorithms for the nearest point problem", Technical Report 80-4, Department of Industrial and Operations Engineering, The University of Michigan, Ann Arbor, Michigan 48109, July, 1980.

14. K. G. Murty, "Convexity of piecewise linear functions", The University of Michigan, Ann Arbor, Michigan 48109 , 1980.

15. C. Perin, "On the polytope of the 1-matching/covering problem", Technical Report No. 81-2, Department of Industrial and Operations Engineering, The University of Michigan, Ann Arbor, Michigan 48109, March, 1981.

16. C. Perin, "The 1-matching/covering problem", Technical Report 81-3, Department of Industrial and Operations Engineering, The University of Michigan, Ann Arbor, Michigan 48109, March, 1981.

17. C. Perin, "User's guide to MATCOV", Technical Report 80-5, Department of Industrial and Operations Engineering, The University of Michigan, Ann Arbor, Michigan 48109, 1980.

18. K. G. Murty and C. Perin, "Parametric algorithm for specified cardinality 1-matching/covering problem", Technical Report 81-6, Department of Industrial and Operations Engineering, The University of Michigan, Ann Arbor, Michigan 48109, 1981.

19. S. J. Chung, "Adjacency on the 0-1 Edge covering problem", Technical Report 80-1, Department of Industrial and Operations Engineering, The University of Michigan, Ann Arbor, Michigan 48109, U.S.A.

20. S. J. Chung, "NP-hard adjacency decision problems on some 0-1 convex polytopes", Technical Report 79-3, Department of Industrial and Operations Engineering, The University of Michigan, Ann Arbor, Michigan 48109, April 1980.

21. A. Gana, "Solving perturbed LCPs with PSD-matrices", Technical Report 81-7, Department of Industrial and Operations Engineering The University of Michigan, Ann Arbor, Michigan 48109, 1981.

PH.D. DISSERTATIONS PARTIALLY SUPPORTED BY THE PROJECT.

22. Y. Fathi, "On the computational complexity of the linear complementarity problem", The University of Michigan, October, 1979.

23. C. Perin, "Matching and edge covering algorithms", The University of Michigan, May, 1980.

24. S. J. Chung, "Structural complexity of adjacency on 0-1 convex polytopes", The University of Michigan, May, 1980.

25. A. Gana, "Studies in the complementarity problem", The University of Michigan, under preparation.

26. P. Carstensen, "Complexity of parametric integer and network flow problems", The University of Michigan, under preparation.

SPOKEN PAPERS PRESENTED AT MEETINGS, CONFERENCES, INVITED LECTURES, ETC.

The principal investigator gave 30 lectures on research performed under this grant at conferences, and at invited seminars held at various universities, between September 1978 to August 1982.

END